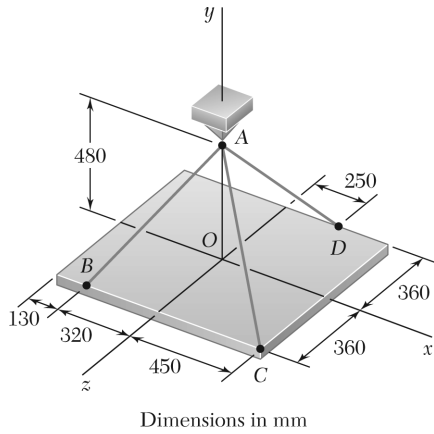


PROBLEM 2.109

A rectangular plate is supported by three cables as shown. Knowing that the tension in cable AC is 60 N, determine the weight of the plate.



SOLUTION

We note that the weight of the plate is equal in magnitude to the force \mathbf{P} exerted by the support on Point A.

$$\Sigma \mathbf{F} = 0: \quad \mathbf{T}_{AB} + \mathbf{T}_{AC} + \mathbf{T}_{AD} + P\mathbf{j} = 0$$

We have:

$$\overline{AB} = -(320 \text{ mm})\mathbf{i} - (480 \text{ mm})\mathbf{j} + (360 \text{ mm})\mathbf{k} \quad AB = 680 \text{ mm}$$

$$\overline{AC} = (450 \text{ mm})\mathbf{i} - (480 \text{ mm})\mathbf{j} + (360 \text{ mm})\mathbf{k} \quad AC = 750 \text{ mm}$$

$$\overline{AD} = (250 \text{ mm})\mathbf{i} - (480 \text{ mm})\mathbf{j} - (360 \text{ mm})\mathbf{k} \quad AD = 650 \text{ mm}$$

Thus:

$$\mathbf{T}_{AB} = T_{AB} \lambda_{AB} = T_{AB} \frac{\overline{AB}}{AB} = \left(-\frac{8}{17}\mathbf{i} - \frac{12}{17}\mathbf{j} + \frac{9}{17}\mathbf{k} \right) T_{AB}$$

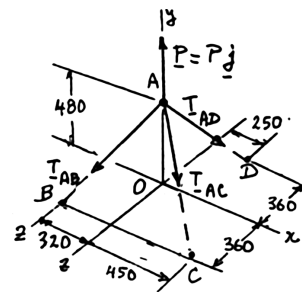
$$\mathbf{T}_{AC} = T_{AC} \lambda_{AC} = T_{AC} \frac{\overline{AC}}{AC} = (0.6\mathbf{i} - 0.64\mathbf{j} + 0.48\mathbf{k}) T_{AC}$$

$$\mathbf{T}_{AD} = T_{AD} \lambda_{AD} = T_{AD} \frac{\overline{AD}}{AD} = \left(\frac{5}{13}\mathbf{i} - \frac{9.6}{13}\mathbf{j} - \frac{7.2}{13}\mathbf{k} \right) T_{AD}$$

Substituting into the Eq. $\Sigma \mathbf{F} = 0$ and factoring \mathbf{i} , \mathbf{j} , \mathbf{k} :

$$\begin{aligned} & \left(-\frac{8}{17}T_{AB} + 0.6T_{AC} + \frac{5}{13}T_{AD} \right) \mathbf{i} \\ & + \left(-\frac{12}{17}T_{AB} - 0.64T_{AC} - \frac{9.6}{13}T_{AD} + P \right) \mathbf{j} \\ & + \left(\frac{9}{17}T_{AB} + 0.48T_{AC} - \frac{7.2}{13}T_{AD} \right) \mathbf{k} = 0 \end{aligned}$$

Free Body A:



PROBLEM 2.109 (Continued)

Setting the coefficient of **i**, **j**, **k** equal to zero:

$$\mathbf{i}: \quad -\frac{8}{17}T_{AB} + 0.6T_{AC} + \frac{5}{13}T_{AD} = 0 \quad (1)$$

$$\mathbf{j}: \quad -\frac{12}{7}T_{AB} - 0.64T_{AC} - \frac{9.6}{13}T_{AD} + P = 0 \quad (2)$$

$$\mathbf{k}: \quad \frac{9}{17}T_{AB} + 0.48T_{AC} - \frac{7.2}{13}T_{AD} = 0 \quad (3)$$

Making $T_{AC} = 60 \text{ N}$ in (1) and (3):

$$-\frac{8}{17}T_{AB} + 36 \text{ N} + \frac{5}{13}T_{AD} = 0 \quad (1')$$

$$\frac{9}{17}T_{AB} + 28.8 \text{ N} - \frac{7.2}{13}T_{AD} = 0 \quad (3')$$

Multiply (1') by 9, (3') by 8, and add:

$$554.4 \text{ N} - \frac{12.6}{13}T_{AD} = 0 \quad T_{AD} = 572.0 \text{ N}$$

Substitute into (1') and solve for T_{AB} :

$$T_{AB} = \frac{17}{8} \left(36 + \frac{5}{13} \times 572 \right) \quad T_{AB} = 544.0 \text{ N}$$

Substitute for the tensions in Eq. (2) and solve for P :

$$\begin{aligned} P &= \frac{12}{17}(544 \text{ N}) + 0.64(60 \text{ N}) + \frac{9.6}{13}(572 \text{ N}) \\ &= 844.8 \text{ N} \end{aligned}$$

Weight of plate = $P = 845 \text{ N}$ ◀